# Design of Robust Digital Controllers for Gas Turbines with Explicit Actuator and Sensor Dynamics

B. Porter* and T. Manganas†
*University of Salford, Salford, England*

**Multi-input multi-output dynamical systems such as gas turbines usually exhibit explicit actuator and sensor dynamics which are often ill-defined. Singular perturbation methods are accordingly used to provide a basis for the design of discrete-time tracking systems incorporating fast-sampling error-actuated digital controllers for multivariable plants with such explicit actuator and sensor dynamics. These general results are illustrated by designing a robust digital controller for the rotational speeds of the low- and high-pressure spools of a typical gas turbine.**

## Introduction

IN recent years, the design of robust digital control systems for multivariable plants has been greatly facilitated by the careful elucidation by Porter[1] of those characteristics of complex multi-input multi-output dynamical systems which determine the achievability of noninteracting control of the various outputs of such systems. This elucidation has led to the development by Porter[1] of powerful design methodologies for discrete-time tracking systems which indicate that noninteracting control is, in general, achievable by the implementation of fast-sampling error-actuated digital controllers in the case of both regular and irregular minimum-phase plants. Thus, in the case of regular plants (i.e., plants with first Markov parameters of maximal rank) noninteracting control is achievable without the need for inner-loop compensators, while in the case of irregular plants (i.e., plants with first Markov parameters of nonmaximal rank) such noninteracting control is also achievable provided only that appropriate inner-loop compensators are introduced.[1]

However, multi-input multi-output dynamical systems such as gas turbines usually exhibit explicit actuator and sensor dynamics which are often ill-defined. The general results of Porter[1] for the design of robust digital control systems for regular multivariable plants with negligible actuator and sensor dynamics have accordingly been extended by Porter and Manganas[2] to embrace dynamical systems with explicit actuator and sensor dynamics. These general results on discrete-time tracking systems are illustrated herein by designing a fast-sampling error-actuated digital controller for the rotational speeds of the low- and high-pressure spools of a typical gas turbine.[3] It is demonstrated by the presentation of the results of extensive design studies that fast noninteracting control of these spool speeds is readily achievable by the implementation of an appropriate fast-sampling digital controller. Furthermore, it is also demonstrated that this fast-sampling digital controller is extremely robust in the face of ill-defined actuator and sensor dynamics by the presentation of simulation results for this gas turbine with third-order actuators and second-order sensors which exhibit minimal performance degradation relative to the design case of first-order actuators and sensors.

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*Professor of Engineering Dynamics and Control, Department of Aeronautical and Mechanical Engineering.

†Research Assistant, Department of Aeronautical and Mechanical Engineering.

## Discrete-Time Tracking Systems

### System Configuration

In general, high-performance discrete-time tracking systems consist of a linear multivariable plant governed on the continuous-time set $\mathfrak{T}=[0,+\infty)$ by state and output equations of the respective forms

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11}, & A_{12} \\ A_{21}, & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} w(t) + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} d(t) \tag{1}$$

$$\dot{w}(t) = -Ew(t) + Eu(t) \tag{2}$$

$$\dot{r}(t) = -Sr(t) + Sy(t) \tag{3}$$

and

$$y(t) = [C_1, C_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{4}$$

together with a fast-sampling error-actuated digital controller governed on the discrete-time set $\mathfrak{T}_T=\{0,T,2T,\dots\}$ by a control-law equation of the form

$$u(kT) = f\{K_1 e(kT) + K_2 z(kT)\} \tag{5}$$

which is required to generate the piecewise-constant actuator input vector $u(t)=u(kT)$, $t\in[kT,\ (k+1)T)$, $kT\in\mathfrak{T}_T$, to cause the plant output vector $y(t)\in\mathfrak{R}^{\ell}$ to track any constant set-point input vector $v(t)\in\mathfrak{R}^{\ell}$ on $\mathfrak{T}_T$ in the sense that

$$\lim_{k\to\infty} \{v(kT) - y(kT)\} = 0 \tag{6}$$

for arbitrary initial conditions where $f=1/T\in\mathfrak{R}^+$ and $T\in\mathfrak{R}^+$ is the sampling period. In Eqs. (1-5), $x_1(t)\in\mathfrak{R}^{n-\ell}$ and $x_2(t)\in\mathfrak{R}^{\ell}$ are the plant state vectors, $w(t)\in\mathfrak{R}^{\ell}$ the actuator state vector, $r(t)\in\mathfrak{R}^{\ell}$ the sensor state vector, $u(t)\in\mathfrak{R}^{\ell}$ the actuator input vector, $A_{11}\in\mathfrak{R}^{(n-\ell)\times(n-\ell)}$, $A_{12}\in\mathfrak{R}^{(n-\ell)\times\ell}$, $A_{21}\in\mathfrak{R}^{\ell x(n-\ell)}$, $A_{22}\in\mathfrak{R}^{\ell\times\ell}$, $B_2\in\mathfrak{R}^{\ell\times\ell}$, $C_1\in\mathfrak{R}^{\ell\times(n-\ell)}$, $C_2\in\mathfrak{R}^{\ell\times\ell}$, $E\in\mathfrak{R}^{\ell\times\ell}$, $S\in\mathfrak{R}^{\ell\times\ell}$, $D_1\in\mathfrak{R}^{(n-\ell)xp}$, $D_2\in\mathfrak{R}^{\ell\times p}$, $d(t)\in\mathfrak{R}^{p}$ the constant disturbance vector, rank $C_2B_2=\ell$, $\sigma(-E)\subset\mathfrak{C}^-$, $\sigma(-S)\subset\mathfrak{C}^-$, $\mathfrak{C}^-$ the open left half-plane, $e(t)=v(t)-r(t)\in\mathfrak{R}^{\ell}$ the error vector, $z\{(k+1)T\}=z(kT)+Te(kT)\in\mathfrak{R}^{\ell}$, $K_1\in\mathfrak{R}^{\ell\times\ell}$, and $K_2\in\mathfrak{R}^{\ell\times\ell}$.

It is evident from Eqs. (1-5) that, in case the actuator matrix $E$ and the sensor matrix $S$ are given by

$$E = fE_0 \tag{7}$$

$$S = fS_0 \tag{8}$$

where $E_0$ and $S_0$ are diagonal matrices with finite positive diagonal elements, such discrete-time tracking systems are governed on $\mathfrak{J}_T$ by closed-loop state and output equations of the respective forms

$$\begin{bmatrix} z\{(k+1)T\} \\ x_1\{(k+1)T\} \\ x_2\{(k+1)T\} \\ w\{(k+1)T\} \\ r\{(k+1)T\} \end{bmatrix} = \begin{bmatrix} I_\ell, & 0, & 0, & 0, & -TI_\ell \\ f\Psi_1K_2, & \Phi_{11}, & \Phi_{12}, & \Phi_{13}, & \Phi_{14}-f\Psi_1K_1 \\ f\Psi_2K_2, & \Phi_{21}, & \Phi_{22}, & \Phi_{23}, & \Phi_{24}-f\Psi_2K_1 \\ f\Psi_3K_2, & \Phi_{31}, & \Phi_{32}, & \Phi_{33}, & \Phi_{34}-f\Psi_3K_1 \\ f\Psi_4K_2, & \Phi_{41}, & \Phi_{42}, & \Phi_{43}, & \Phi_{44}-f\Psi_4K_1 \end{bmatrix} \begin{bmatrix} z(kT) \\ x_1(kT) \\ x_2(kT) \\ w(kT) \\ r(kT) \end{bmatrix} + \begin{bmatrix} TI_\ell \\ f\Psi_1K_1 \\ f\Psi_2K_1 \\ f\Psi_3K_1 \\ f\Psi_4K_1 \end{bmatrix} v(kT) + \begin{bmatrix} 0 \\ \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \end{bmatrix} d(kT) \tag{9}$$

and

$$y(kT) = [0, C_1, C_2, 0, 0] \begin{bmatrix} z(kT) \\ x_1(kT) \\ x_2(kT) \\ w(kT) \\ r(kT) \end{bmatrix} \tag{10}$$

where

$$\begin{bmatrix} \Phi_{11}, & \Phi_{12}, & \Phi_{13}, & \Phi_{14} \\ \Phi_{21}, & \Phi_{22}, & \Phi_{23}, & \Phi_{24} \\ \Phi_{31}, & \Phi_{32}, & \Phi_{33}, & \Phi_{34} \\ \Phi_{41}, & \Phi_{42}, & \Phi_{43}, & \Phi_{44} \end{bmatrix} = \exp\left\{ \begin{bmatrix} A_{11}, & A_{12}, & 0, & 0 \\ A_{21}, & A_{22}, & B_2, & 0 \\ 0, & 0, & -fE_0, & 0 \\ fS_0C_1, & fS_0C_2, & 0, & -fS_0 \end{bmatrix} T \right\} \tag{11}$$

$$\begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{bmatrix} = \int_0^T \exp\left\{ \begin{bmatrix} A_{11}, & A_{12}, & 0, & 0 \\ A_{21}, & A_{22}, & B_2, & 0 \\ 0, & 0, & -fE_0, & 0 \\ fS_0C_1, & fS_0C_2, & 0, & -fS_0 \end{bmatrix} t \right\} \times \begin{bmatrix} 0 \\ 0 \\ fE_0 \\ 0 \end{bmatrix} dt \tag{12}$$

and

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \end{bmatrix} = \int_0^T \exp\left\{ \begin{bmatrix} A_{11}, & A_{12}, & 0, & 0 \\ A_{21}, & A_{22}, & B_2, & 0 \\ 0, & 0, & -fE_0, & 0 \\ fS_0C_1, & fS_0C_2, & 0, & -fS_0 \end{bmatrix} t \right\} \times \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \end{bmatrix} dt \tag{13}$$

Therefore, provided that the eigenvalues of the closed-loop plant matrix in Eq. (9) lie in the open unit disk $\mathfrak{D}^-$,

$$\lim_{k\to\infty} e(kT) = \lim_{k\to\infty} \{v(kT) - r(kT)\} \qquad = \lim_{k\to\infty} \{v(kT) - y(kT)\} = 0 \tag{14}$$

so that set-point tracking occurs together with disturbance rejection.

**System Analysis**

The transfer function matrix relating the plant output vector to the set-point input vector of the closed-loop discrete-time tracking system governed by Eqs. (9) and (10) is clearly

$$G(z)=[0,C_1,C_2,0,0]\begin{bmatrix} zI_\ell-I_\ell, & 0, & 0, & 0, & TI_\ell \\ -f\Psi_1K_2, & zI_{n-\ell}-\Phi_{11}, & -\Phi_{12}, & -\Phi_{13}, & -\Phi_{14}+f\Psi_1K_1 \\ -f\Psi_2K_2, & -\Phi_{21}, & zI_\ell-\Phi_{22}, & -\Phi_{23}, & -\Phi_{24}+f\Psi_2K_1 \\ -f\Psi_3K_2, & -\Phi_{31}, & -\Phi_{32}, & zI_\ell-\Phi_{33}, & -\Phi_{34}+f\Psi_3K_1 \\ -f\Psi_4K_2, & -\Phi_{41}, & -\Phi_{42}, & -\Phi_{43}, & zI_\ell-\Phi_{44}+f\Psi_4K_1 \end{bmatrix}^{-1}\begin{bmatrix} TI_\ell \\ f\Psi_1K_1 \\ f\Psi_2K_1 \\ f\Psi_3K_1 \\ f\Psi_4K_1 \end{bmatrix} \tag{15}$$

and the fast-sampling tracking performance of this system can accordingly be elucidated by invoking the results of Porter and Shenton[4] from the singular perturbation analysis of transfer function matrices. These results indicate that as $f\rightarrow\infty$ the closed-loop transfer function matrix $G(z)$ assumes the asymptotic form

$$\begin{aligned}\Gamma(z)=\{&(z-1+p_2)[z(1-p_1/\alpha_0)\\&+p_1+p_1/\alpha_0-1]\}\{(z-1)^3I_\ell\\&+(z-1)^2[(p_1+p_2)I_\ell+\alpha_0\theta_2C_2B_2K_1]\\&+(z-1)[p_1p_2I_\ell+\{p_1\theta_1+p_1\alpha_0\theta_2\\&+p_2(1-p_1/\alpha_0)\}C_2B_2K_1]+p_1p_2C_2B_2K_1\}^{-1}C_2B_2K_1\end{aligned} \tag{16}$$

where

$$p_1=1-e^{-\alpha_0} \tag{17}$$

$$p_2=1-e^{-\delta_0} \tag{18}$$

$$\theta_1=\delta_0\{1/2!-(\alpha_0+\delta_0)/3!+(\alpha_0^2+\alpha_0\delta_0+\delta_0^2)/4!\ldots\} \tag{19}$$

and

$$\theta_2=\delta_0\{1/3!-(\alpha_0+\delta_0)/4!+(\alpha_0^2+\alpha_0\delta_0+\delta_0^2)/5!\ldots\} \tag{20}$$

in case

$$E_0=\alpha_0I_\ell \tag{21}$$

and

$$S_0=\delta_0I_\ell \tag{22}$$

Furthermore, the "slow" modes $\mathcal{Z}_s$ of the tracking system correspond as $f\rightarrow\infty$ to the poles $\mathcal{Z}_1\cup\mathcal{Z}_2$ of the "slow" transfer function matrix where

$$\mathcal{Z}_1=\{z\in\mathbb{C}: |zI_\ell-I_\ell+TK_1^{-1}K_2|=0\} \tag{23}$$

and

$$\mathcal{Z}_2=\{z\in\mathbb{C}: |zI_{n-\ell}-I_{n-\ell}-TA_{11}+TA_{12}C_2^{-1}C_1|=0\} \tag{24}$$

while the "fast" modes $\mathcal{Z}_f$ of the tracking system correspond as $f\rightarrow\infty$ to the poles $\mathcal{Z}_3$ of the "fast" transfer function matrix where

$$\begin{aligned}\mathcal{Z}_3=\{z\in\mathbb{C}: |&(z-1)^3I_\ell+(z-1)^2[(p_1+p_2)I_\ell\\&+\alpha_0\theta_2C_2B_2K_1]+(z-1)[p_1p_2I_\ell+\{p_1\theta_1+p_1\alpha_0\theta_2\\&+p_2(1-p_1/\alpha_0)\}C_2B_2K_1]+p_1p_2C_2B_2K_1|=0\}\end{aligned} \tag{25}$$

Finally, the "slow" modes $\mathcal{Z}_s$ are not poles of the asymptotic transfer function matrix $\Gamma(z)$ since the "slow" modes $\mathcal{Z}_1$ become asymptotically uncontrollable as $f\rightarrow\infty$ and the "slow" modes $\mathcal{Z}_2$ become asymptotically unobservable as $f\rightarrow\infty$.[2]

#### System Synthesis

It is evident from Eqs. (9) and (10) that set-point tracking, together with disturbance rejection, will occur in the sense of Eq. (14) provided only that

$$\mathcal{Z}_s\cup\mathcal{Z}_f\subset\mathfrak{D}^- \tag{26}$$

where $\mathfrak{D}^-$ is the open unit disk. In view of Eqs. (23-25), the "slow" and "fast" modes will satisfy the tracking requirement [Eq. (26)] for sufficiently high sampling frequencies if the controller matrices $K_1$ and $K_2$ are chosen such that $\mathcal{Z}_1\subset\mathfrak{D}^-$ for sufficiently small sampling periods and $\mathcal{Z}_3\subset\mathfrak{D}^-$ in the case of minimum-phase plants for which the set of transmission zeros[5]

$$\mathcal{Z}_t=\{s\in\mathbb{C}: |sI_{n-\ell}-A_{11}+A_{12}C_2^{-1}C_1|=0\}\subset\mathbb{C}^- \tag{27}$$

where $\mathbb{C}^-$ is the open left half-plane since it is then immediately obvious from Eq. (24) that $\mathcal{Z}_2\subset\mathfrak{D}^-$ for sufficiently small sampling periods. Furthermore, if $K_1$ is chosen such that

$$C_2B_2K_1=\Sigma=\sigma I_\ell \tag{28}$$

where $\sigma\in\mathbb{R}^+$, it follows from Eq. (16) that the transfer function matrix $G(z)$ of the discrete-time tracking system assumes the asymptotic diagonal form

$$\begin{aligned}\Gamma(z)=\{&(z-1+p_2)[z(1-p_1/\alpha_0)+p_1+p_1/\alpha_0-1]\sigma\}\\&\times\{(z-1)^3+(z-1)^2[p_1+p_2+\alpha_0\theta_2\sigma]+(z-1)[p_1p_2\\&+\{p_1\theta_1+p_1\alpha_0\theta_2+p_2(1-p_1/\alpha_0)\}\sigma]+p_1p_2\sigma\}^{-1}I_\ell\end{aligned} \tag{29}$$

as $f\rightarrow\infty$, and therefore that increasingly noninteracting tracking will occur as $f\rightarrow\infty$.

#### System Performance Specification

It is evident from Eq. (29) that, as $f\rightarrow\infty$, each channel of the multi-input multi-output closed-loop tracking system behaves as a third-order discrete-time system and therefore that the required time-domain performance can be specified by selecting the controller tuning parameter $\sigma$. Now, the continuous-time characteristic equation of a third-order system with one real root

$$s_1=-\eta\omega_n \tag{30}$$

and two conjugate complex roots

$$s_{2,3}=-\nu\omega_n\pm i\omega_n\sqrt{1-\nu^2} \tag{31}$$

clearly has the form

$$s^3+s^2(\eta+2\nu)\omega_n+s(1+2\nu\eta)\omega_n^2+\eta\omega_n^3=0 \tag{32}$$

where $\nu$ and $\omega_n$ are the damping ratio and natural frequency of the complex roots, respectively, and $\eta\in\mathbb{R}^+$ the ratio of the real root to the modulus of the complex roots. These continuous-time roots obviously map under the transformation $z=e^{sT}$

into the discrete-time roots

$$z_1 = e^{-\eta\xi} \tag{33}$$

and

$$z_{2,3} = e^{-\nu\xi}\{\cos(\xi\sqrt{1-\nu^2}) \pm i\sin(\xi\sqrt{1-\nu^2})\} \tag{34}$$

where $\xi = T\omega_n$.

In view of Eqs. (25) and (28), it is evident that each channel of the closed-loop tracking system is governed by the discrete-time characteristic equation

$$(z-1)^3 + (z-1)^2[p_1+p_2+\alpha_0\theta_2\sigma] + (z-1)[p_1p_2 + \{p_1\theta_1+p_1\alpha_0\theta_2+p_2(1-p_1/\alpha_0)\}\sigma] + p_1p_2\sigma = 0 \tag{35}$$

Therefore, it follows from Eqs. (33-35) that the roots $\{z_1, z_2, z_3\}$ given by Eqs. (33) and (34) will coincide with the roots of Eq. (35) in case

$$\eta = -\frac{1}{\xi}\ln[1-2\{e^{-\nu\xi}\cos(\xi\sqrt{1-\nu^2})-1\}-p_1-p_2-\alpha_0\theta_2\sigma] \tag{36}$$

$$\sigma = [2\{e^{-\nu\xi}\cos(\xi\sqrt{1-\nu^2})-1\}+p_1+p_2] \times [e^{-2\nu\xi}-1-2\{e^{-\nu\xi}\cos(\xi\sqrt{1-\nu^2})-1\}] \div (p_1p_2-\alpha_0\theta_2[e^{-2\nu\xi}-1-2\{e^{-\nu\xi}\cos(\xi\sqrt{1-\nu^2})-1\}]) \tag{37}$$

and

$$2[2\{e^{-\nu\xi}\cos(\xi\sqrt{1-\nu^2})-1\}+p_1+p_2+1][e^{-\nu\xi}\cos(\xi\sqrt{1-\nu^2})-1]-e^{-2\nu\xi}+1+p_1p_2 = -\sigma[p_1\theta_1+p_1\alpha_0\theta_2 + p_2(1-p_1/\alpha_0)+2\alpha_0\theta_2\{e^{-\nu\xi}\cos(\xi\sqrt{1-\nu^2})-1\}] \tag{38}$$

provided that

$$0 < 1-2[e^{-\nu\xi}\cos(\xi\sqrt{1-\nu^2})-1]-p_1-p_2-\alpha_0\theta_2\sigma \leq 1 \tag{39}$$

and

$$p_1p_2-\alpha_0\theta_2[e^{-2\nu\xi}-1-2\{e^{-\nu\xi}\cos(\xi\sqrt{1-\nu^2})-1]\neq 0 \tag{40}$$

Thus, by specifying $\nu$ in Eqs. (36-38), the parameters $\sigma$, $\xi$, and $\eta$ are determined as functions of $\alpha_0$ and $\delta_0$. Indeed, Eqs. (36-38) show that the controller tuning parameter $\sigma$ must be "matched" to both the actuator and sensor time constants for the closed-loop tracking system to exhibit the required time-domain performance.

## Digital Controller for Gas Turbine

The relevance of these general results to the design of robust digital controllers for gas turbines can be conveniently illustrated by designing a fast-sampling error-actuated digital controller for the gas turbine governed on $\mathfrak{Z}$ by the respective state and output equations[3]

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1.268, & -0.04528 \\ 1.002, & -1.957 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1.498, & 951.5 \\ 8.52, & 1240 \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d(t) \tag{41}$$

$$\begin{bmatrix} \dot{w}_1(t) \\ \dot{w}_2(t) \end{bmatrix} = \begin{bmatrix} -100, & 0 \\ 0, & -100 \end{bmatrix} \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix} + \begin{bmatrix} 100, & 0 \\ 0, & 100 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \tag{42}$$

$$\begin{bmatrix} \dot{r}_1(t) \\ \dot{r}_2(t) \end{bmatrix} = \begin{bmatrix} -200, & 0 \\ 0, & -200 \end{bmatrix} \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} + \begin{bmatrix} 200, & 0 \\ 0, & 200 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} \tag{43}$$

and

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1, & 0 \\ 0, & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{44}$$

where the state variables are

$x_1(t)$ = high-pressure-spool speed

$x_2(t)$ = low-pressure-spool speed

$w_1(t)$ = jet-pipe-nozzle area

$w_2(t)$ = fuel-flow rate

$r_1(t)$ = high-pressure-spool speed sensor state

$r_2(t)$ = low-pressure-spool speed sensor state (45)

the inputs are

$u_1(t)$ = demanded jet-pipe-nozzle area,

$u_2(t)$ = demanded fuel-flow rate (46)

the disturbance $d(t)$ represents a change of altitude, and the outputs $y_1(t)$ and $y_2(t)$ are the two spool speeds. These variables are all expressed as perturbations from an operating point, and the slow jet-pipe-nozzle actuator of time constant 0.1 s (used by McMorran[3]) has been phase-lead-compensated[6] so that its effective time constant becomes 0.01 s, while the sensors (neglected by McMorran[3]) have time constants of 0.005 s.

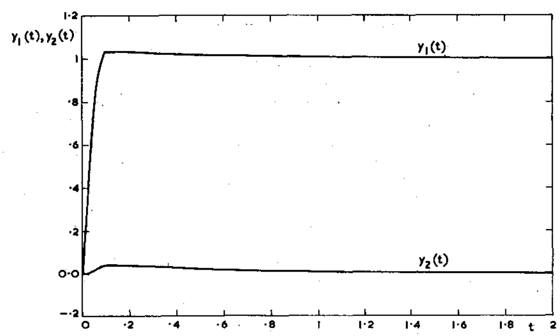
It can be readily verified from Eqs. (41) and (44) that $\mathbb{Z}_t = \emptyset$ and that the first Markov parameter

$$C_2B_2 = \begin{bmatrix} 1.498, & 951.5 \\ 8.52, & 1240.0 \end{bmatrix} \tag{47}$$

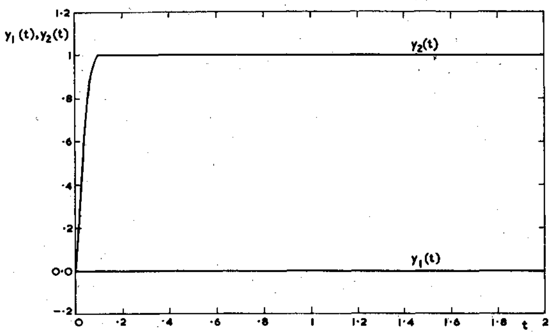
is of full rank. In case $T = 0.01$ s, $\nu = 0.8$, and $K_2 = 2K_1$, it follows from Eqs. (36-38) that $\eta = 5.98458$, $\xi = 0.418572$, and $\sigma = 0.211468$, and from Eqs. (5), (28), and (47) that the corresponding fast-sampling error-actuated digital controller is governed on $\mathfrak{Z}_T$ by the control-law equation

$$\begin{bmatrix} u_1(kT) \\ u_2(kT) \end{bmatrix} = \begin{bmatrix} -4.19601, & 3.21977 \\ 0.028831, & -0.005069 \end{bmatrix} \begin{bmatrix} e_1(kT) \\ e_2(kT) \end{bmatrix} + \begin{bmatrix} -8.39202, & 6.43954 \\ 0.057661, & -0.010138 \end{bmatrix} \begin{bmatrix} z_1(kT) \\ z_2(kT) \end{bmatrix} \tag{48}$$

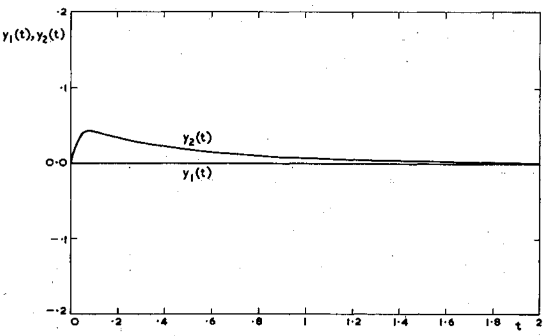
Furthermore, it is evident from Eqs. (23-25) that the resulting sets of assigned asymptotic closed-loop characteristic roots are




a) $v(t) = [1,0]^T$, $d(t) = 0$ (first-order actuators and sensors).

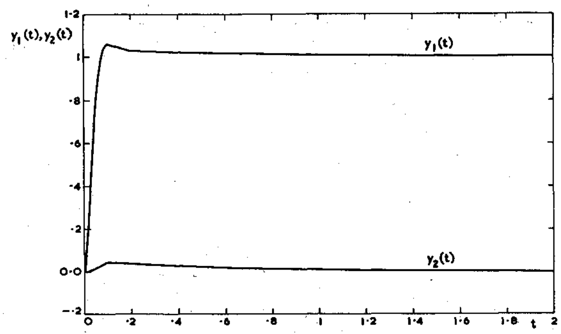



b) $v(t) = [0,1]^T$, $d(t) = 0$ (first-order actuators and sensors).

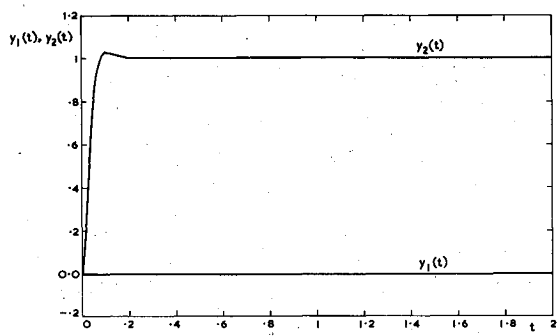



c) $v(t) = [0,0]^T$, $d(t) = 1$ (first-order actuators and sensors).

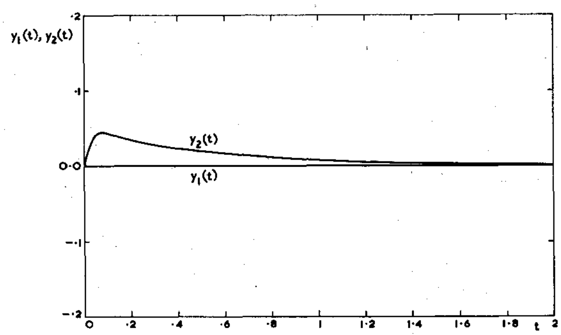
**Fig. 1 Closed-loop step responses.**




a) $v(t) = [0,1]^T$, $d(t) = 0$ (third-order actuators and second-order sensors).




b) $v(t) = [0,1]^T$, $d(t) = 0$ (third-order actuators and second-order sensors).




c) $v(t) = [0,0]^T$, $d(t) = 1$ (third-order actuators and second-order sensors).

**Fig. 2 Closed-loop step responses.**

$$\mathcal{Z}_1 = \{0.98,\ 0.98\}$$

and

$$\mathcal{Z}_3 = \{0.081677,\ 0.692996 \pm 0.177795i,\ 0.081677,\ 0.692996 \pm 0.177795i\} \tag{49}$$

while it is found by direct computation that the corresponding sets of actual closed-loop characteristic roots are

$$\mathcal{Z}_1 = \{0.979862,\ 0.979296\}$$

and

$$\mathcal{Z}_3 = \{0.082255,\ 0.696463 \pm 0.16944i,\ 0.082180,\ 0.693581 \pm 0.173218i\} \tag{50}$$

The corresponding step-response characteristics are shown in Figs. 1a, 1b, and 1c when $[v_1(t),\ v_2(t)]^T = [1,0]^T$ and $d(t) = 0$, $[v_1(t),\ v_2(t)]^T = [0,1]^T$ and $d(t) = 0$, and $[v_1(t),\ v_2(t)]^T = [0,0]^T$ and $d(t) = 1$, respectively, and indicate that fast noninteracting control of the spool speeds is achieved together with excellent disturbance rejection.

The robustness of this control system is demonstrated in Figs. 2 in which the step-response characteristics are shown when the fast-sampling digital controller governed on $\mathfrak{J}_T$ by the control-law equation [Eq. (48)] is applied to the gas turbine with third-order actuators each with transfer function

$$g_a(s) = \frac{4.08 \times 10^7}{(s+120)(s+300+500i)\,(s+300+500i)} \tag{51}$$

and second-order sensors each with transfer function

$$g_s(s) = \frac{9 \times 10^4}{(s+180)\,(s+500)} \tag{52}$$

These results show that the excellent closed-loop set-point tracking performance, together with disturbance rejection, of the closed-loop system is not affected by the presence of unmodeled fast actuator and sensor modes or by inaccuracies in the estimation of the dominant first-order modes of the actuators and sensors.

## Conclusion

Singular perturbation methods have been used to provide a basis for the design of discrete-time tracking systems incorporating fast-sampling error-actuated digital controllers for multivariable plants with explicit actuator and sensor dynamics. It has been shown that the resulting design methodology greatly facilitates the determination of controller matrices which ensure that the closed-loop tracking behavior becomes increasingly noninteracting as the sampling frequency is increased, and that specified closed-loop performance can be readily achieved by "matching" the controller tuning parameter to the actuator and sensor time constants. These general results have been illustrated by designing a robust digital controller for the rotational speeds of the low- and high-pressure spools of a typical gas turbine. The excellent set-point tracking and disturbance-rejection characteristics of this controller are greatly superior to the corresponding characteristics of the controller designed by Seraji[7] for the same gas turbine using conventional pole-assignment techniques.

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